Meeting the Formwork Requirements for a Challenging Structure

Casting crisp details in cathedral's complex reliquary walls

BY ERIC PETERSON

Challenges: One-sided formwork where curved and battered architectural concrete reliquary walls intersected at acute angles, radial and annular control lines for form setting, and carefully constructed reveals to hide joints in the forming panels.



The Cathedral of Christ the Light in Oakland, CA, is the central cathedral of the Roman Catholic Diocese of Oakland. Construction of the award-winning structure began in 2005 and was completed in 2008, nearly 20 years after the Loma Prieta earthquake damaged the Oakland Diocese's Cathedral of St. Francis de Sales beyond repair. Unique aspects of the new cathedral include:

- Seismic isolation of the superstructure using frictionpendulum isolation bearings isolators to protect the cathedral structure from damage caused by a maximum capable earthquake;
- Curved and battered perimeter concrete reliquary walls that function structurally and as exposed architectural surfaces;
- A tension-supported glued laminated (glulam) timber system for a sanctuary superstructure supported at the base by the reliquary walls and at the top by a steel compression ring containing an ocular skylight;
- Two massive windows bounded by the compression ring at the top, the glulam system on the sides, and the perimeter concrete walls at the bottom; and
- A unitized fritted glass curtain wall that surrounds the glulam superstructure.

Each of these aspects is worthy of a stand-alone article, but the major emphasis here is on forming the concrete support structure and the architectural concrete reliquary walls.

TOP TO BOTTOM OVERVIEW

As shown in Fig. 1, the major superstructure element forming the Cathedral space is the glass and glulam laminated timber roof that rises above architectural concrete perimeter walls. In plan (Fig. 2), the inner and outer concrete perimeter walls form two parallel circular arcs divided into six sectors by passageways directed toward the altar. Along with a concrete ceiling and floor slab, the walls create a box beam structure supporting the curved glulam timber ribs and louvers of the vaulted superstructure (Fig. 3). In addition to being battered, the curved box beam walls are interspersed with niches, narrow windows, and other openings including the passageways (Fig. 4).

The box beams span about 45 ft (13.7 m) with 20 friction pendulum base isolators located directly beneath the reliquary walls (Fig. 2 and 3). The sanctuary floor slab also serves as a structural diaphragm, below which 16 more isolators are positioned. The 36 isolators provide for 30 in. (760 mm) of displacement capacity in anticipation of the maximum capable earthquake.



Working drawings of the formwork described here are available with the online version of this article at www.concreteinternational.com



Fig. 1: The glued laminated timber superstructure rises above curved and battered perimeter reliquary walls of architectural concrete



Fig. 2: Inner and outer concrete perimeter walls form two parallel circular arcs divided into six sectors by passageways directed toward the altar (*Image courtesy of ©SOM*)

The mausoleum level beneath the box beam reliquary walls and the seismic isolation system contains a series of cruciform-shaped reinforced concrete bearing walls, 18 ft (5.5.m) high, that bear on a 2.5 to 4 ft (0.76 to 1.2 m) thick mat foundation and a perimeter basement wall foundation. Crypt walls between the cruciform walls serve as lateral supports to prevent buckling, and tops of all the walls are interconnected with a 6 in. (150 mm) thick continuous reinforced concrete diaphragm—the ceiling for the mausoleum. Short pedestals were cast on this diaphragm and centered above the cruciform wall intersections to support the seismic isolators below a two-way reinforced concrete flat slab that, with toppings applied, is the sanctuary floor. The lightweight superstructure combined with the heavy foundation and



Fig. 3: Along with a concrete ceiling and floor slab, the walls create a box beam structure supporting the curved glulam timber ribs and louvers of the vaulted superstructure. Note that the perimeter beams beneath the box beams were about 6 ft (1.8 m) wide and narrowed the space between the mausoleum ceiling slab and the soffit of the sanctuary floor (1 ft = 0.305 m) (Image courtesy of @SOM)



Fig. 4: The curved, battered box beam reliquary walls contain rustications, niches, fixtures, and openings



reliquary walls resulted in a low center of gravity, thus reducing vertical acceleration in a seismic event.

WHY SELF-CONSOLIDATING CONCRETE?

The decision to use self-consolidating concrete (SCC) was based on several considerations, including:

- Reinforcing bar congestion, especially near the intersections of elements;
- Difficulty in inserting and withdrawing internal vibrators when consolidating concrete for the battered reliquary walls;
- The need for sharp, crisp details at the many acute corners and recesses in the reliquary walls;
- Desire for a dense, nearly uniform appearance of the reliquary walls; and
- Limited accessibility for some of the placements, thus requiring the concrete to flow into place.

Webcor had not placed SCC on any previous projects, so a secondary consideration was advancement of our own construction skills in using new technologies. This could be done painlessly because there were no architectural requirements for the mausoleum walls, but using SCC there as well provided an opportunity to experiment with the wall forming system, formwork materials, SCC mixture, and casting techniques.

FORMING THE SUPPORT STRUCTURE

After completing mausoleum wall placements, the 6 in. (150 mm) thick diaphragm ceiling slab was formed and placed. Short pedestals were cast on this slab to support the isolators below the sanctuary floor. The next step—forming, shoring, reshoring, and then removing forms



Fig. 5: A very long set of graphite-coated opposing wood wedges with square-cut ends supported many of the junior beams that shored formwork for the sanctuary floor. The interior area in which these were used was almost inaccessible, but the wedges could be removed by sliding with a hydraulic ram, thus releasing the vertical load and permitting form stripping (1 in. = 25.4 mm; 1 ft = 0.305 m)

for the two-way sanctuary floor slab—was especially challenging. About 3.25 ft (1 m) separated the top of the mausoleum level ceiling slab and the bottom of the sanctuary slab. To reduce time spent by workers crawling on their hands and knees, we bought several creepers (rolling platforms used mainly by mechanics when working under cars) and dollies to move both workers and formwork.

Form panels for the cathedral floor slab were placed on shortened versions of commercially available drop-head shores. This made form stripping easy, with the only critical operation being positioning of the reshores for the mausoleum ceiling. With fairly long spans and only a 6 in. (150 mm) mausoleum ceiling thickness, reshores had to be placed directly beneath the shores for the cathedral slab to avoid overstressing the mausoleum ceiling slab.

Removing formwork below perimeter beams located beneath the reliquary walls was much more difficult. These beams were about 6 ft (1.8 m) wide, with soffits about 13 in. (0.33 m) above the ceiling slab. The form for the beam soffit was supported by aluminum joists resting on steel junior beams (see glossary), leaving very little space for supporting the junior beams.

Some of the junior beams could be supported with very stubby sand jacks (see glossary), but these jacks were not suitable for use on the almost inaccessible interior of the area between the two reliquary walls. In those areas, we used a very long set of graphite-coated opposing wood wedges with square cut ends (Fig. 5). One wedge assembly could support two junior beams, and by calculating the vertical load that would compress the wedges, we could also calculate the longitudinal force required to slide out one of the wedges with a hydraulic ram, thus releasing the vertical load and permitting form stripping.

To demonstrate the viability of this device, we set up two junior beams on a set of wedges, then placed the equivalent load in the form of concrete deadmen on top of the entire system. The object of the test was two-fold: to verify the required jacking force predicted by the calculation and to also verify that the blunt end of the wedge would not crush when force was applied. The wedges functioned well, saving Webcor both time and money.

ARCHITECTURAL FORMWORK

Each of the reliquary wall surfaces forms a portion of a cone, with the outer wall surface forming a portion of an upright cone, and the inner wall forming a portion of an inverted cone. Wall bases on each side of the cathedral were defined by 13 radial control lines spaced at 8.081 degrees and two annular control lines with



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radii of 108 ft 9-3/4 in. and 121 ft 2-1/4 in. (33.17 and 36.94 m). The formwork system selected was provided by Atlas Construction Supply (Fig. 6). Where battered walls intersected at acute angles, one-sided formwork was needed because wall ties could not be used in such locations (Fig. 6 and 7). The selection of SCC also had a major impact on the formwork because it had to sustain full liquid head pressure with practically zero deflection or movement. Joints, seams, bulkheads, and form bottoms had to be leak- proof and form-facing materials had to produce an acceptable finish when used with SCC.

One of the first steps in the design of the formwork was the production of accurate shop drawings for each wall elevation. These drawings not only had to account for the

curvature of the panel but also had to accurately locate all of the reveals, plywood seam locations, openings, and other features attached to or embedded in the walls. These features included electrical receptacles, recessed signs, plaques, and life safety devices.

To prevent deflection of the formwork during placement, the studs and wales were closely spaced and the entire form was sheathed with two layers of 3/4 in. (19 mm) plywood; a base course of medium-density overlay (MDO) plywood and a finish layer of high-density overlay (HDO) plywood, the specified form facing material. HDO plywood is available in multiple grades, with one- or two-step sanded face veneers directly beneath phenolic overlays of varying weights. A good quality HDO plywood is very durable and produces a concrete surface without the wood grain texture typically obtained with standard plyform.

Several steps were taken to keep the surface smooth and prevent discoloration of the concrete. Using two layers of plywood allowed the face sheets to be backscrewed so that no fasteners were used from the finish side and also allowed the plywood joints to be offset between the two layers, preventing grout leakage that could cause discoloration at the plywood face joints.

Reveal strips were fabricated using medium-density fiber board (MDF) because it was inexpensive, easy to mill, and easily bent to follow the curved form panels. Completely sealing the MDF was essential to prevent it from absorbing water and discoloring the concrete around the reveals. Sealant tooled to a small radius was installed between the reveal strip and the form face and at the corners of recesses to further limit grout leakage and provide a smooth transition into the reveal.



Fig. 6: Formwork for the reliquary walls included interior braces and ties for uplift. Where wall ties couldn't be used, bracket assemblies were used to support exterior strongbacks. Working drawings of the formwork used on this project are available with the online version of this article at www.concreteinternational.com (1 in. = 25.4 mm; 1 ft = 0.305 m; 1 kip = 4.45 kN)



Fig. 7: To form the curved reliquary walls, radius-cut timbers were attached to horizontal timber beams (generally, $2-1/2 \times$ 5-1/2 in. [64 x 140 mm] laminated veneer lumber) that were attached to the battered strongbacks. Wall ties couldn't be used where walls intersected at acute angles, so we used one-sided formwork supported by the green brace assemblies shown here

PROJECT CREDITS

Client: Catholic Cathedral Corporation of the East Bay Design Architect: Skidmore, Owings & Merrill Architect of Record: Kendall/Heaton Associates Structural Engineer: Skidmore, Owings & Merrill General Contractor: Webcor Builders Concrete Contractor: Webcor Concrete Concrete Supplier: Central Concrete Formwork Supplier: Atlas Construction Supply The only continuous vertical reveals in the walls occurred at the radial control lines. This determined the gang panel joint locations. To maintain alignment of two gang panel faces, workers routed $1/8 \ge 3/8$ in. ($3 \ge 10$ mm) grooves into the vertical edges of the face sheets to permit insertion of a $1/8 \ge 3/4$ in. ($3 \ge 10$ mm) spline. This kept the faces flush and ensured that any leakage from behind the vertical reveal would be contained and not cause discoloration. All of the steps taken in the lead-side forming operations are outlined in the sidebar on page 48.

Braces to the floor slab were used to hold the battered outside wall forms in place, while the interior forms were braced against one another (Fig. 6). After the interior forms were in place, a casting platform was constructed at the level of the top of the wall. Buoyant pressure of the SCC on the inclined outer face of the wall produced a calculated 8 kip (36 kN) vertical uplift at the bottom of each strongback. This uplift was resisted by turnbuckle chain binders held by steel angle iron cross beams (Fig. 6 and 8).

After initial concrete curing, the formwork for the walls was carefully removed to avoid damaging the HDO plywood faces and cleaned so it could be reused. The formwork for the ceiling was then put in place. Because the sides of the beams at the ceiling level would be visible, it was important that the ceiling appear to be cast monolithically with the walls. Therefore, the same SCC used for the walls was also used for the ceiling, with the construction joint between the two placements concealed in a horizontal reveal. An integral curb at the top of the structure required concrete with a stiffer consistency—a semiflowable SCC— so the slump flow of the mixture was reduced to 16 to 18 in. (410 to 460 mm) from the 24 to 27 in. (610 to 690 mm) used for the walls. This was achieved by adjusting the admixture dosage without making any other changes.



Fig. 8: Buoyant pressure of the SCC on the inclined outer face of the walls produced a vertical uplift at the bottom of each strongback. This uplift was resisted by turnbuckle chain binders held by steel cross beams (fabricated from rolled angles) tied into the sanctuary floor slab

To verify that the proposed formwork and casting procedure would produce the desired appearance of the architectural concrete, a full-scale mockup of the reliquary wall was cast. The mockup included a reliquary wall with a niche, an intersecting interior wall, an intersecting end wall, and a portion of the ceiling. It proved to be valuable based on the number of lessons we learned from constructing it.

Although we were able to prove that our basic formwork concept was solid, the mockup revealed that we didn't understand the geometry of the structure as well as we thought we did and that more definition was required from the designer. Some of the reinforcement had to be refabricated due to a drawing interpretation error. Because of some slight displacement of the formwork during casting of the mockup, we learned that the formwork had to be modified to better resist overturning. Although the actual structure had two opposing walls that could be internally braced, the importance of the bracing was made quite clear during the casting of the mockup section. Finally, the mockup confirmed calculations showing that the walls did not have to be reshored, which had concerned us because they were inclined by 10 degrees.

TRIAL RUN IN THE MAUSOLEUM

Although SCC was not necessary for the mausoleum level walls, they provided an opportunity to test the formwork, troubleshoot any problems that were encountered, work on solutions for problems that developed, and familiarize the crew with placing SCC. Among the problems encountered during these early placements were segregation, inconsistent workability, blocking at congested areas, difficulty maintaining a flowable mixture for a sufficient amount of time, foaming, surface air voids, and pumping difficulties. The major lessons learned from these early placements included:

- Using a hydration stabilizing admixture to delay set for 4 to 6 hours kept the SCC fluid long enough to place the walls for a complete sector with a single pump, resulting in the elimination of pour lines and improved pumpability;
- Adding a steel tremie pipe at the end of the pump line and keeping the tip constantly embedded at least 6 in. (150 mm) minimized bugholes in the formed surfaces;
- Limiting concrete flow within the forms to about 40 ft (12 m) maintained consistent coarse aggregate distribution; and
- Testing every truckload of SCC for slump flow before it was placed in the pump hopper helped assure us that once in the form, the concrete would perform as needed. This testing procedure was our best guarantee that we wouldn't discover problems too late—after concrete was in the pump hopper.

REASONS FOR SUCCESS

Construction of the reliquary walls was costly, but the expense was expected because of the complex formwork for the artistically curved and battered walls. This part of the concrete work was completed successfully in large part due to careful thought and planning by the joint design and construction team. The walls, however, are only a portion of the exemplary craftsmanship exhibited in the cathedral by all of the trades. Other work of note includes the decorative concrete topping of the cathedral floor slabs and ornate wood and metalwork in the roof and fritted glass curtain walls. Also noteworthy is the "Omega Window" that rises behind the altar and uses more than 94,000 holes with 100 different diameters cut through aluminum window panels to reproduce a 12th-century depiction of Christ from the façade of Chartres Cathedral in France. More information and photos can be found at www.som.com and www.christthelightcathedral.org.

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Sequence of lead-side form operations:

- Concrete outline drawings approved;
- Formwork drawings verified;
- Typical outside panels set;
- End wall and passageway panels set;
- Geometric accuracy verified by surveyors;
- Joint offset limitations checked and met;
- Recesses, openings, and special features installed;
- Reveals installed;
- Corners, reveals, and form bottoms caulked and sealed;
- Form release agent applied and wiped down;
- Architectural and quality control reviews;
- Form protection installed prior to placing reinforcing; and
- Release formwork for reinforcing steel placement.

Glossary of terms

Readers may be unfamiliar with some of the terms used in this article. Brief definitions are given below:

drop-head shore—a temporary support that allows slab forming panels and panel support beams to be lowered and removed while the support continues to carry load.

junior beam—an open-web steel beam initially developed by the Junior Company of Los Angeles, CA.

lead-side form—the first side to be erected during the assembly of a wall form.

reliquary—a shrine that stores relics.

sand jack—a sand-filled casing in which the sand can be confined or released as needed to support or lower a plunger.



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He is especially skilled in the area of coordinate geometry, formwork design, and detailing and is a member of ACI Committee 117, Tolerances.















































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